Multilevel Models for Longitudinal Data

Track 1

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Longitudinal Data Structure

- Person-level dataset [multivariate, wide-dataset]
  - Each subject has one row [or record]
  - Repeated measures appear as additional variables
  - No explicit "time" variable

- Person-period dataset [univariate, long-dataset]
  - A subject identifier
  - A time indicator
  - Outcome variable[s]
  - Predictor variable[s]

Example Data

Data comes from the *National Youth Survey* (Raudenbush & Chan, 1992)

Five waves, ages 11 - 15

- TOL, Tolerance of deviant behavior
  ($1 =$ very wrong, $4 =$ not wrong at all)
- MALE, 1 for male, 0 for female
- EXP, self reported exposure to deviant behavior at age 11
  ($0 =$none, $4 =$all).

“Person-level” Data Set

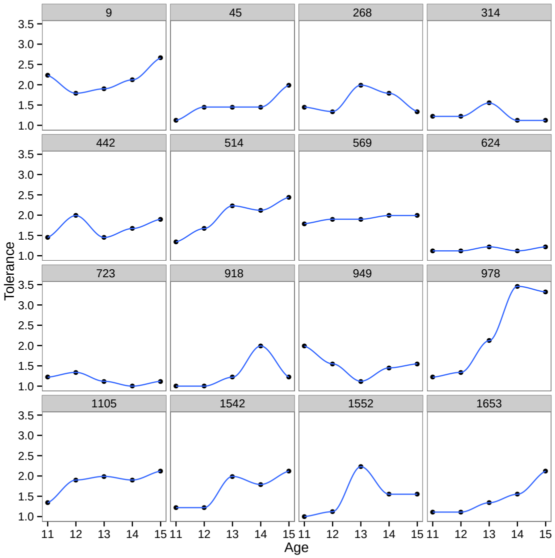
ID	TOL11	TOL12	TOL13	TOL14	TOL15	MALE	EXP
9	2.23	1.79	1.90	2.12	2.66	0	1.54
45	1.12	1.45	1.45	1.45	1.99	1	1.16
268	1.45	1.34	1.99	1.79	1.34	1	0.90
314	1.22	1.22	1.55	1.12	1.12	0	0.81
442	1.45	1.99	1.45	1.67	1.90	0	1.13
514	1.34	1.67	2.23	2.12	2.44	1	0.90
569	1.79	1.90	1.90	1.99	1.99	0	1.99
624	1.12	1.12	1.22	1.12	1.22	1	0.98
723	1.22	1.34	1.12	1.00	1.12	0	0.81
918	1.00	1.00	1.22	1.99	1.22	0	1.21
949	1.99	1.55	1.12	1.45	1.55	1	0.93
978	1.22	1.34	2.12	3.46	3.32	1	1.59
1105	1.34	1.90	1.99	1.90	2.12	1	1.38
1542	1.22	1.22	1.99	1.79	2.12	0	1.44
1552	1.00	1.12	2.23	1.55	1.55	0	1.04
1653	1.11	1.11	1.34	1.55	2.12	0	1.25

“Person-period” Data Set

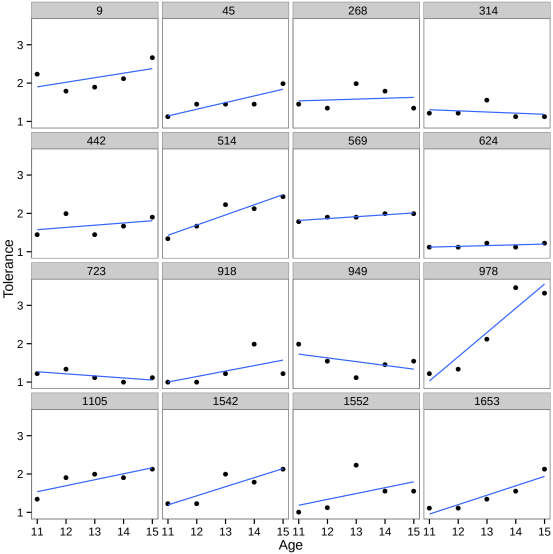
ID	MALE	EXP	AGE	TOL
9	0	1.54	11	2.23
9	0	1.54	12	1.79
9	0	1.54	13	1.90
9	0	1.54	14	2.12
9	0	1.54	15	2.66
45	1	1.16	11	1.12
45	1	1.16	12	1.45
45	1	1.16	13	1.45
45	1	1.16	14	1.45
45	1	1.16	15	1.99
.	.	.	.	.
.	.	.	.	.
1653	0	1.25	11	1.11
1653	0	1.25	12	1.11
1653	0	1.25	13	1.34
1653	0	1.25	14	1.55
1653	0	1.25	15	2.12

Exploring Longitudinal Data

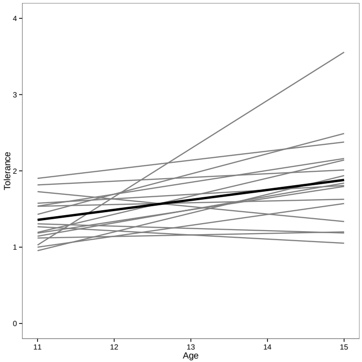
Exploring Longitudinal Data: Non-parametric Summaries



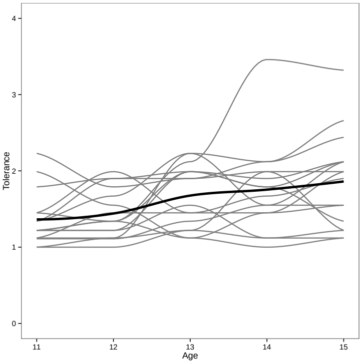
Exploring Longitudinal Data: Fitted OLS Trajectories



Exploring Longitudinal Data

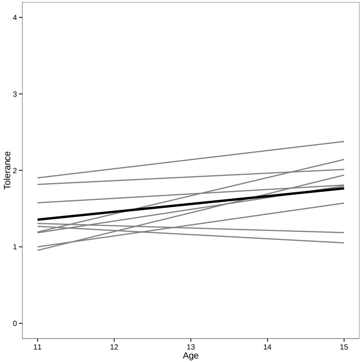


(a) Fitted OLS

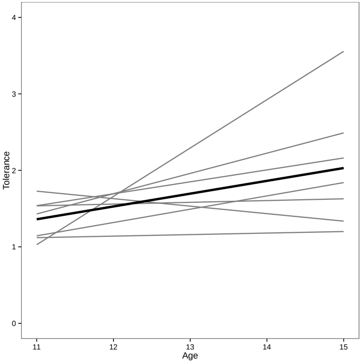


(b) Smooth non-parametric

Exploring Longitudinal Data, by Male/Female

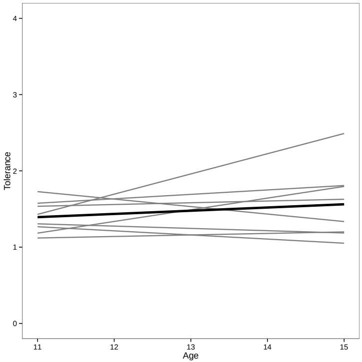


(c) Female

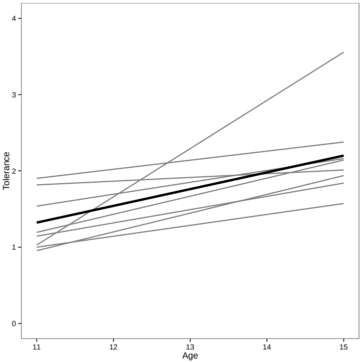


(d) Male

Exploring Longitudinal Data, by Exposure (High > 1.145)



(e) Low Exposure



(f) High Exposure

The Multilevel Model for Change

The first example is limited to:

- Linear change model
- Time-structured data set
- Evaluation of one time-invariant dichotomous predictor

Example Data

- Data comes from Burchinal et al. (1997)
- 103 African-American infants born into low-income families
- At 6 months old, approximately half the sample ($n = 53$) were randomly assigned to participate in an intensive early intervention program designed to enhance cognitive functioning
- The remaining children ($n = 45$) were assigned to a control group
- Infants assessed 12 times between ages 6 and 96 months

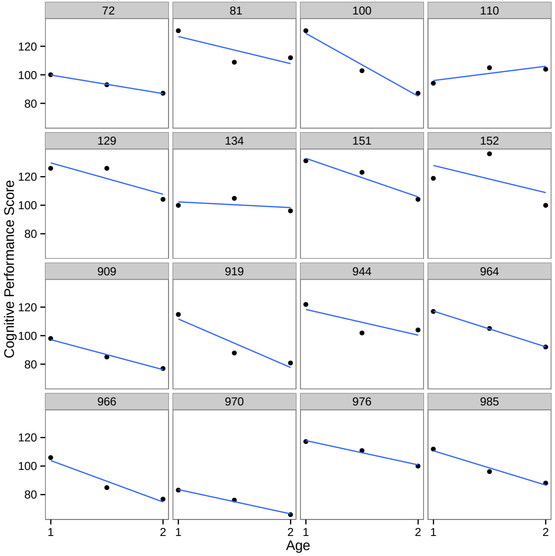
Example Data

- 3 waves of data–each child has three records
- *AGE* (in years) is the child's age at each assessment (1, 1.5, or 2)
- *COG* is the child's cognitive performance score at each assessment
- *PROGRAM* is a dichotomous covariate, 1= treatment and 0= control

Example Data

ID	COG	AGE	PROGRAM
68	103	1.0	1
68	119	1.5	1
68	96	2.0	1
70	106	1.0	1
70	107	1.5	1
70	96	2.0	1
.	.	.	.
.	.	.	.
984	106	1.0	0
984	89	1.5	0
984	99	2.0	0
985	112	1.0	0
985	96	1.5	0
985	88	2.0	0

Empirical Growth Plots: Fitted OLS Trajectories



The Multilevel Model for Change

$$Y_{ij} = \pi_{0i} + \pi_{1i}(AGE_{ij} - 1) + \epsilon_{ij}$$
$$\pi_{0i} = \gamma_{00} + \gamma_{01}(PROGRAM_i) + \zeta_{0i}$$
$$\pi_{1i} = \gamma_{10} + \gamma_{11}(PROGRAM_i) + \zeta_{1i},$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{10}^2 & \sigma_1^2 \end{bmatrix} \right)$$

- The Level-1 Submodel
  - Describes how each person changes over time
  - Research questions about within-person change
- The Level-2 Submodel
  - Describes how these changes differ across people.
  - Research questions about between-person change

The Level-1 Submodel

$$Y_{ij} = [\pi_{0i} + \pi_{1i}(AGE_{ij} - 1)] + [\epsilon_{ij}]$$

Where,

- Y_{ij} represents the value of COG for child i at time j
  - i runs from 1 to 103
  - j runs from 1 to 3
- Brackets distinguish between the structural part and the stochastic part of the model
  - The structural part parallels the concept of "true score"
  - The stochastic part parallels the concept of "measurement error"

The Structural Part of the Level-1 Submodel

$$Y_{ij} = [\pi_{0i} + \pi_{1i}(AGE_{ij} - 1)] + [\epsilon_{ij}]$$

Our hypothesis about the shape of each subject's *true trajectory of change* over time

- π_{0i} represents child i's true initial cognitive performance (at age 1).
  - π_{01} is the intercept for child 1
  - π_{02} is the intercept for child 2
- π_{1i} represents the slope of the postulated individual change trajectory
  - If π_{1i} is positive, subject i's outcome increases over time

The Stochastic Part of the Level-1 Submodel

$$Y_{ij} = [\pi_{0i} + \pi_{1i}(AGE_{ij} - 1)] + [\epsilon_{ij}]$$

- ϵ_{ij} represents the effect of random error associated with individual i at time j
- ϵ_{ij} is unobserved so we must make assumptions about the distribution of level 1 residuals from occasion to occasion and from person to person.

The Stochastic Part of the Level-1 Submodel

$$\epsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\epsilon^2)$$

- "Classical" assumptions specify residuals as independently and identically distributed ("iid"), with homoscedastic variance across occasions and individuals.
- Classical assumptions may not hold with longitudinal data as residuals may be autocorrelated and heteroscedastic over time.

The Level-2 Submodel

$$\pi_{0i} = [\gamma_{00} + \gamma_{01}(PROGRAM_i)] + [\zeta_{0i}]$$
$$\pi_{1i} = [\gamma_{10} + \gamma_{11}(PROGRAM_i)] + [\zeta_{1i}]$$

Where,

- π_{0i} and π_{1i} represents the level-1 change parameters–initial status and linear growth
- Brackets distinguish between the structural part and the stochastic part of the model
  - The structural part parallels the concept of “true score”
  - The stochastic part parallels the concept of “measurement error”

The Structural Part of the Level-2 Submodel

$$\pi_{0i} = [\gamma_{00} + \gamma_{01}(PROGRAM_i)] + [\zeta_{0i}]$$

$$\pi_{1i} = [\gamma_{10} + \gamma_{11}(PROGRAM_i)] + [\zeta_{1i}]$$

Where,

- γs represent the level-2 regression parameters–known as *fixed effects*
- Fixed effects capture inter individual differences in the true change trajectory
- Interpret fixed effects as a *prototypical individual*:
  - γ_{00} represents the *average initial status* for children not enrolled in the treatment ($PROGRAM = 0$)
  - γ_{10} represents the *average annual growth* for children not enrolled in the treatment ($PROGRAM = 0$)
  - $\gamma_{00} + \gamma_{01}$ represents the *average initial status* for children enrolled in the treatment ($PROGRAM = 1$)
  - $\gamma_{10} + \gamma_{11}$ represents the *average annual growth* for children enrolled in the treatment ($PROGRAM = 1$)

The Stochastic Part of the Level-2 Submodel

$$\pi_{0i} = [\gamma_{00} + \gamma_{01}(PROGRAM_i)] + [\zeta_{0i}]$$

$$\pi_{1i} = [\gamma_{10} + \gamma_{11}(PROGRAM_i)] + [\zeta_{1i}]$$

Where,

- ζ represent the residuals–what remained *unexplained by the fixed effects*
- Less interested in values of ζ than in the population summaries of the variances σ_0^2 and σ_1^2, and covariance σ^2

The Stochastic Part of the Level-2 Submodel

$$\begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

Standard assumption about the level-2 residuals:

- Bivariate normal distribution
- Mean of zero
- Unknown variance and covariance parameters

Model Results

	Parameter	Estimate	ase	95% CI
Fixed Effects				
π_{0i}, Initial status	γ_{00}, Intercept	107.84	2.04	$[103.85, 111.83]$
	γ_{01}, PROGRAM	6.86	1.88	$[1.54, 12.17]$
π_{0i}, Rate of change	γ_{10}, Intercept	-21.13	1.88	$[-24.83, -17.44]$
	γ_{11}, PROGRAM	5.27	2.51	$[0.35, 10.19]$
Variance Components				
Level 1:	σ_{ϵ}^2	74.76		
Level 2:	σ_0^2	123.97		
	σ_1^2	10.10		
	σ_{01}	-35.38		

Interpreting Fixed Effects

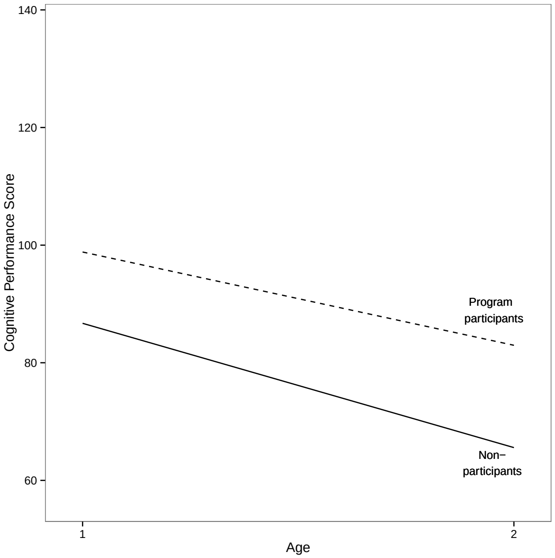
$$\hat{\pi}_{0i} = 107.84 + 6.86(PROGRAM_i)$$

$$\hat{\pi}_{1i} = -21.13 + 5.27(PROGRAM_i)$$

Where,

- $107.84 =$ Initial status (*COG* at age=1) for the average *nonparticipant*
- $6.86 =$ Difference in initial status for the average *participant*
- $-21.13 =$ Annual rate of change for the average *nonparticipant*
- $5.27 =$ Difference in annual rate of change for the average *participant*

Fitted Change Trajectories in COG



Single Parameter Tests for Fixed Effects

Testing the statistical significance of fixed effects is similar to multiple regression where $H_0 : \gamma = 0$ and $H_1 : \gamma \neq 0$

Test this hypothesis for each fixed effect by computing a z-statistic:

$$z = \frac{\hat{\gamma}}{ase(\hat{\gamma})}$$

Interpreting Variance Components

$$\sigma_\epsilon^2 = 74.76$$

$$\begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} = \begin{bmatrix} 123.97 & -35.38 \\ -35.38 & 10.10 \end{bmatrix}$$

Where,

- Level-1 residual variance, σ_ϵ^2, summarizes within-person variability
- Level-2 variance components summarize between-person variability in change trajectories
- Single-parameter tests of significance for variance components can be highly inconsistent

Extending the Multilevel Model for Change

Extending the Multilevel Model for Change

- The composite formulation
- Unconditional means model and unconditional growth model
- Model building strategies

Adolescent Alcohol Use Data

- Curran, Stice, and Chassin (1997) collected 3 waves of data
- Time-structured data set of 82 adolescents beginning at age 14.
  - *ALCUSE*, the level of alcohol consumption during the *previous* year
  - *AGE*, the age of the child at the time of data collection
  - *PEER*, a measure of alcohol use among the adolescent's peers
  - *COA*, a dichotomous covariate, indicating if the adolescent is a child of an alcoholic (1=yes, 0=no)

ALCUSE and *PEER* are generated by computing the square root of the sum of the participants' responses across each variable's constituent items.

Composite Specification of the Multilevel Model for Change

$$\begin{aligned} Y_{ij} &= \pi_{0i} + \pi_{1i} TIME_{ij} + \epsilon_{ij} \\ \pi_{0i} &= \gamma_{00} + \gamma_{01} COA_i + \zeta_{0i} \\ \pi_{1i} &= \gamma_{10} + \gamma_{11} COA_i + \zeta_{1i} \end{aligned}$$

$$\begin{aligned} Y_{ij} &= \pi_{0i} + \pi_{1i} TIME_{ij} + \epsilon_{ij} \\ &= (\gamma_{00} + \gamma_{01} COA_i + \zeta_{0i}) + (\gamma_{10} + \gamma_{11} COA_i + \zeta_{1i}) TIME_{ij} + \epsilon_{ij} \\ &= \gamma_{00} + \gamma_{10} TIME_{ij} + \gamma_{01} COA_i + \gamma_{11} (COA_i \times TIME_{ij}) + \\ &\quad \zeta_{0i} + \zeta_{1i} TIME_{ij} + \epsilon_{ij} \end{aligned}$$

The Unconditional Means Model

$$Y_{ij} = \gamma_{00} + \zeta_{0i} + \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \zeta_{0i} \sim N(0, \sigma_0^2)$$

- Describes and partitions the outcome *variation*.
- Assumes the true individual change trajectory for person i is flat, sitting at elevation $\gamma_{00} + \zeta_{0i}$, or π_{0i}.
- Average (*grand mean*) elevation, across everyone, is γ_{00}.
- Partions the total outcome variation by within-person, σ_ϵ^2 and between-person, σ_0^2.

Model Results

	Model 1	Model 2	Model 3	Model 4	Model 5
Fixed Effects					
γ_{00}, Initial status	0.922	0.651	0.316	−0.317	−0.314
	(0.096)	(0.105)	(0.131)	(0.148)	(0.146)
γ_{01}, COA		1.88	0.743	0.579	0.571
			(0.195)	(0.162)	(0.146)
γ_{02}, PEER				0.694	0.695
				(0.112)	(0.111)
γ_{10}, Rate of change		0.271	0.293	0.429	0.425
		(0.062)	(0.084)	(0.114)	(0.106)
γ_{11}, COA			−0.049	0.014	
			0.125	(0.125)	
γ_{12}, PEER				−0.150	−0.151
				(0.086)	(0.085)
Variance Components					
σ_ϵ^2	0.562	0.337	0.337	0.337	0.337
σ_0^2	0.564	0.624	0.488	0.241	0.241
σ_1^2		0.151	0.151	0.139	0.139
σ^2		−0.068	−0.059	−0.006	−0.006

The Intraclass Correlation Coefficient, ICC

$$\rho = \frac{\sigma_0^2}{\sigma_0^2 + \sigma_\epsilon^2}$$

$$= \frac{0.564}{0.562 + 0.564} = \frac{0.564}{1.126} = 0.501$$

- Describes the proportion of total variance that lies between people.
- Also know as the *error autocorrelation coefficient.*

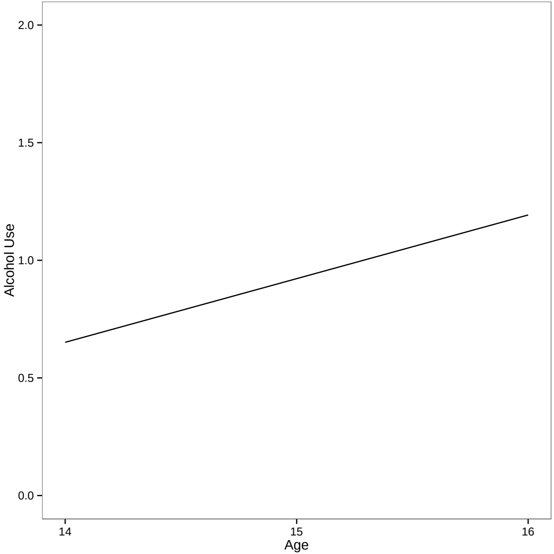
The Unconditional Growth Model

$$Y_{ij} = \gamma_{00} + \gamma_{10} TIME_{ij} + \zeta_{0i} + \zeta_{1i} TIME_{ij} + \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

- Describes the unconditional initial status and rate of change for the population.
- $\gamma_{00} + \zeta_{0i}$ represents the interindividual initial status
- $\gamma_{10} + \zeta_{1i}$ represents the interindividual rate of change
- σ_ϵ^2 summarizes each person's data around his/her linear change trajectory
- σ_0^2 and σ_0^2 summarize between-person variability in initial status and rates of change.

The Unconditional Growth Model Graphically



Pseudo R^2– Understanding the effect of *TIME*

$$\frac{\sigma^2_{\epsilon_{Model1}} - \sigma^2_{\epsilon_{Model2}}}{\sigma^2_{\epsilon_{Model1}}} = \frac{0.562 - 0.337}{0.562} = 0.4004$$

- 40% of the with-in person variation in *ALCUSE* is systematically associated with linear *TIME*.

The Unconditional Growth Model Covariance

$$\begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

$$\hat{\rho}_{\pi_0 \pi_1} = \hat{\rho}_{01} = \frac{\sigma_{01}}{\sqrt{\sigma_0^2 \, \sigma_0^2}} = \frac{-0.068}{\sqrt{(0.624)(0.151)}} = -0.22$$

- The linear relationship between *ALCUSE* at age 14, γ_{00} and rate of change in *ALCUSE* between age 14 and 16, γ_{10} is weakly negative.

A Taxonomy Of Statistical Models

- A *taxonomy* of models is a "systematic sequence of models that, as a set, address your research question" (Singer & Willett, 2003, p. 105).

- Distinguish between *control* predictors and *question* predictors.
  - In our example, we will assume our research questions focuses on *COA*.
  - *PEER* is used as a control.

The Uncontrolled Effects of *COA*

$$
\begin{aligned}
Y_{ij} = & \pi_{0i} + \pi_{1i} TIME_{ij} + \epsilon_{ij} \\
\pi_{0i} &= \gamma_{00} + \gamma_{01} COA_i + \zeta_{0i} \\
\pi_{1i} &= \gamma_{10} + \gamma_{10} COA_i + \zeta_{1i}
\end{aligned}
$$

$$
Y_{ij} = \gamma_{00} + \gamma_{01} COA_i + \gamma_{10} TIME_{ij} + \gamma_{10}(TIME_{ij} COA_i) + \zeta_{0i} + \zeta_{1i} TIME_{ij} + \epsilon_{ij}
$$

$$
\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)
$$

- γ_{01} describes the difference in the level of *ALCUSE* at age 14 for children with and without alcoholic parents.
- γ_{11} describes the impact of *COA* on the rate of change in *ALCUSE* between ages 14 and 16.

The Uncontrolled Effects of *COA*

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Fixed Effects					
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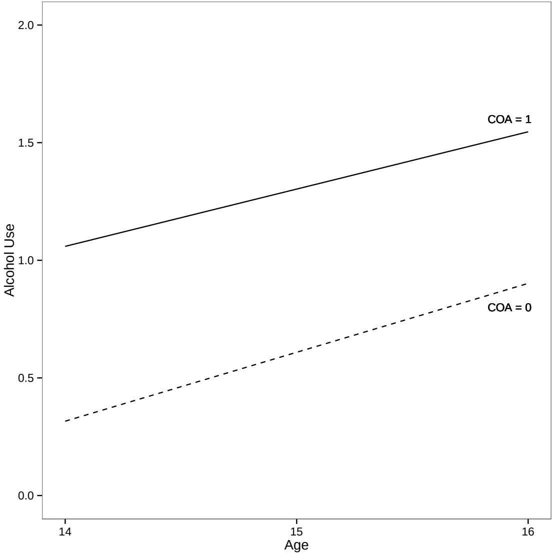
σ_0^2	0.564	0.624	0.488	0.241	0.241
σ_1^2		0.151	0.151	0.139	0.139
σ_{01}		−0.068	−0.059	−0.006	−0.006

The Uncontrolled Effects of *COA* Graphically



The Uncontrolled Effects of *COA*

	Model 1	Model 2	Model 3	Model 4	Model 5
Pseudo R^2 Statistics					
R^2_ϵ		0.400	0.000	0.000	0.000
R^2_0			0.219	0.501	0.000
R^2_1			0.004	0.076	0.000
Goodness-of-fit					
Deviance	670.156	636.611	621.203	588.691	588.703
df	3	6	8	10	9
BIC	686.672	669.643	665.245	643.744	638.251

Comparing Models Using Deviance Statistics

- Comparing models using *deviance statistics* is a more robust approach than using single parameter tests
  1. Superior statistical properties.
  2. Permits composite tests on several parameters.
  3. "Reserves the reservoir of Type I error" (Singer & Willett, 2003, p. 116).
- FML tests all parameters while REML tests only variance components.

$$\text{Deviance} = -2[\ell_{\text{current model}} - \ell_{\text{saturated model}}]$$

- ℓ is the log-likelihood, a byproduct of ML estimation–the larger the ℓ (closer to 0) the better the fit.
- The saturated model is a general mode that fits the data perfectly.
- Deviance quantifies how much worse the current model fits the data compared to the best possible model.

Comparing Models Using Deviance Statistics

$$\text{Deviance} = -2[\ell_{\text{current model}} - \ell_{\text{saturated model}}]$$
$$= -2[\ell_{\text{current model}} - 0]$$
$$= -2\ell_{\text{current model}}$$

- $\ell_{\text{saturated model}}$ = 0 because the probability that the model will perfectly fit the data is 1 ($log(1) = 0$).
- -2 because standard normal theory assumptions say that comparing nested models has a known distribution.

Comparing Models Using Deviance Statistics

Deviance-based Hypothesis Tests:

- Data set must be unchanged across models.
- The former model must be nested within the latter model.
- Compute the number of additional constraints imposed.
- ΔD is distributed asymptotically as a χ^2 distribution. with $d.f. =$ the number of independent constraints imposed.

$$\Delta D = \text{Deviance}_{\text{Reduced Model}} - \text{Deviance}_{\text{Full Model}}$$

$$\begin{aligned}\Delta D &= \text{Deviance}_{\text{Model 2}} - \text{Deviance}_{\text{Model 3}} \\ &= 636.611 - 621.203 = 15.408\end{aligned}$$

15.408 exceeds the χ^2 .001 critical value at 2 $d.f.$ (13.816), allowing us to reject the null hypothesis that γ_{01} and γ_{11} are simultaneously 0.

The Controlled Effects of *COA*

$$
\begin{aligned}
Y_{ij} =& \gamma_{00} + \gamma_{01} COA_i + \gamma_{02} PEER_i + \gamma_{10} TIME_{ij} + \\
& \gamma_{11}(TIME_{ij} COA_i) + \gamma_{12}(TIME_{ij} PEER_i) + \zeta_{0i} + \zeta_{1i} TIME_{ij} + \epsilon_{ij}
\end{aligned}
$$

$$
\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)
$$

- γ_{02} describes the impact of peer alcohol use on the level of *ALCUSE* at age 14 for children, controlling for *COA*.
- γ_{12} describes the impact of peer alcohol use on the rate of change in *ALCUSE* between ages 14 and 16, controlling for *COA*..

The Controlled Effects of *COA*

	Model 1	Model 2	Model 3	Model 4	Model 5
Fixed Effects					
γ_{00}, Initial status	0.922	0.651	0.316	−0.317	−0.314
	(0.096)	(0.105)	(0.131)	(0.148)	(0.146)
γ_{01}, COA		1.88	0.743	0.579	0.571
			(0.195)	(0.162)	(0.146)
γ_{02}, PEER				0.694	0.695
				(0.112)	(0.111)
γ_{10}, Rate of change		0.271	0.293	0.429	0.425
		(0.062)	(0.084)	(0.114)	(0.106)
γ_{11}, COA			−0.049	0.014	
			0.125	(0.125)	
γ_{12}, PEER				−0.150	−0.151
				(0.086)	(0.085)
Variance Components					
σ^2_ϵ	0.562	0.337	0.337	0.337	0.337
σ^2_0	0.564	0.624	0.488	0.241	0.241
σ^2_1		0.151	0.151	0.139	0.139
σ_{01}		−0.068	−0.059	−0.006	−0.006

The Controlled Effects of *COA*

	Model 1	Model 2	Model 3	Model 4	Model 5
Pseudo R^2 Statistics					
R^2_ϵ		0.400	0.000	0.000	0.000
R^2_0			0.219	0.501	0.000
R^2_1			0.004	0.076	0.000
Goodness-of-fit					
Deviance	670.156	636.611	621.203	588.691	588.703
df	3	6	8	10	9
BIC	686.672	669.643	665.245	643.744	638.251

Final Model for Controlled Effects of *COA*

$$Y_{ij} = \gamma_{00} + \gamma_{01} COA_i + \gamma_{02} PEER_i + \gamma_{10} TIME_{ij} + \gamma_{12}(TIME_{ij} PEER_i) + \zeta_{0i} + \zeta_{1i} TIME_{ij} + \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{10} & \sigma_1^2 \end{bmatrix} \right)$$

The Controlled Effects of *COA*

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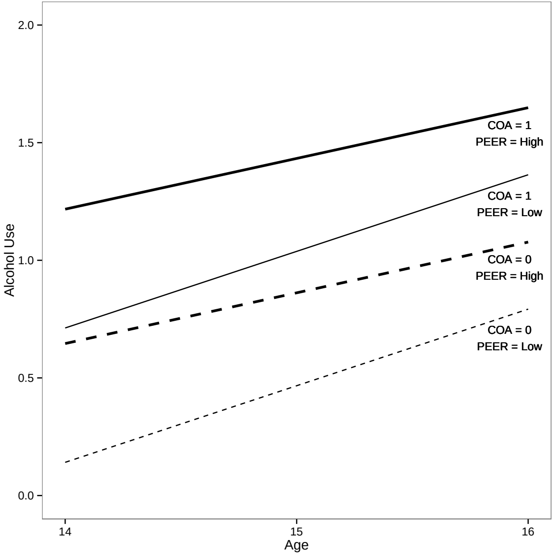
The Controlled Effects of *COA* Graphically



The Controlled Effects of *COA*

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Deviance Tests When Model Trimming

$$\Delta D = \text{Deviance}_{\text{Reduced Model}} - \text{Deviance}_{\text{Full Model}}$$

$$\Delta D = \text{Deviance}_{\text{Model 5}} - \text{Deviance}_{\text{Model 4}}$$

$$= 588.703 - 588.691 = 0.012$$

0.012 does not exceed the χ^2 .001 critical value at 1 $d.f.$ (3.841). We are unable to reject the null hypothesis that γ_{11} is 0.

For Further Study

Hedeker, D. & Gibbons, R.D., (2006) Longitudinal Data Analysis. Hoboken, Wiley.

Singer, J. & Willett, J. (2003) Applied Longitudinal Analysis. New York, Oxford University Press.

Skrondal, A. & Rabe-Hesketh, S. (2004) Generalized Latent Variable Modeling. Boca Raton, Chapman & Hall/CRC.

Thank you!